CSE 114A

Introduction to Functional Programming

Intro to Haskell

Based on course materials developed by Nadia Polikarpova and Owen Arden

What is Haskell?

- A **typed**, **lazy**, **purely functional** programming language
  - Haskell = λ-calculus +
    - Better syntax
    - Types
    - Built-in features
      - Booleans, numbers, characters
      - Records (tuples)
      - Lists
      - Recursion
      - …

Why Haskell?

- Haskell programs tend to be *simple* and *correct*
- ***Quicksort in Haskell***

```
sort []     = []
sort (x:xs) = sort ls ++ [x] ++ sort rs
  where
    ls      = [ l | l <- xs, l <= x ]
    rs      = [ r | r <- xs, x <  r ]
```

- ***Goals***
  - Understand the above code
  - Understand what **typed**, **lazy**, and **purely functional** means (and why you care)

Haskell vs λ-calculus: Programs

- A program is an expression (not a sequence of statements)

- It evaluates to a value (it does not perform actions)
  - λ:
    ```
    (\x -> x) apple     -- =~> apple
    ```
  - Haskell:
    ```
    (\x -> x) "apple"  -- =~> "apple"
    ```

Haskell vs λ-calculus: Functions

- **Functions are first-class values:**
  - can be ***passed as arguments*** to other functions
  - can be ***returned as results*** from other functions
  - can be ***partially applied*** (arguments passed *one at a time*)

```
(\x -> (\y -> x (x y))) (\z -> z + 1) 0   -- =~> 2
```

- **BUT:** unlike λ-calculus, not everything is a function!

Haskell vs λ-calculus: top-level bindings

- Like in Elsa, we can name terms to use them later
- **Elsa:**

```
let T    = \x y -> x
let F    = \x y -> y

let PAIR = \x y -> \b -> ITE b x y
let FST  = \p -> p T
let SND  = \p -> p F

eval fst:
 FST (PAIR apple orange)
 =~> apple
```

Haskell vs λ-calculus: top-level bindings

- Like in Elsa, we can name terms to use them later
- **Haskell:**

```
haskellIsAwesome = True
pair = \x y -> \b -> if b then x else y
fst = \p -> p haskellIsAwesome
snd = \p -> p False

-- In GHCi:
> fst (pair "apple" "orange")   -- “apple"
```

- **The names are called top-level variables**
- **Their definitions are called top-level bindings**

Syntax: Equations and Patterns

- You can define function bindings using **equations**:

```
pair x y b = if b then x else y -- pair = \x y b -> ...
fst p      = p True             -- fst = \p -> ...
snd p      = p False            -- snd = \p -> …
```

Syntax: Equations and Patterns

- A single function binding can have *multiple* equations with different **patterns** of parameters:

```
pair x y True  = x  -- If 3rd arg matches True,
                    -- use this equation;
pair x y False = y  -- Otherwise, if 3rd arg matches
                    -- False, use this equation.
```

- The first equation whose pattern matches the actual arguments is chosen
- For now, a pattern is:
  - a variable (matches any value)
  - or a value (matches only that value)

Syntax: Equations and Patterns

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```
pair x y True  = x   -- If 3rd arg matches True,
                     -- use this equation;
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                     -- False, use this equation.
```

- Same as:

```
pair x y True  = x   -- If 3rd arg matches True,
                     -- use this equation;
pair x y b     = y   -- Otherwise use this equation.
```

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- Same as:

```
pair x y True  = x   -- If 3rd arg matches True,
                     -- use this equation;
pair x y _     = y   -- Otherwise use this equation.
```

QUIZ: Pair

Which of the following definitions of pair is incorrect? *

A.
```haskell
pair x y = \b -> if b then x else y
```

B.
```haskell
pair x = \y b -> if b then x else y
```

C.

```haskell
pair x _ True  = x
pair _ y _     = y
```

D.

```haskell
pair x y b     = x
pair x y False = y
```

E. all of the above

http://tiny.cc/cse116-pair-ind

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D.

```haskell
pair x y b     = x
pair x y False = y
```

E. all of the above

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Equations with guards

- An equation can have multiple guards (Boolean expressions):

```
cmpSquare x y  |  x > y*y   =  "bigger :)"
               |  x == y*y  =  "same :|"
               |  x < y*y   =  "smaller :("
```

- Same as:

```
cmpSquare x y  |  x > y*y   =  "bigger :)"
               |  x == y*y  =  "same :|"
               |  otherwise =  "smaller :("
```

Recursion

- Recursion is built-in, so you can write:

```
sum n = if n == 0
          then 0
          else n + sum (n - 1)
```

- Or you can write:

```
sum 0 = 0
sum n = n + sum (n - 1)
```

Scope of variables

- Top-level variables have global scope

```
message = if haskellIsAwesome  -- this var defined below
            then "I love CSE 114A"
            else "I'm dropping CSE 114A”
haskellIsAwesome = True
```

- Or you can write:

```
-- What does f compute?
f 0 = True
f n = g (n - 1) -- mutual recursion!
g 0 = False
g n = f (n - 1) -- mutual recursion!
```

- Answer: `f` is `isEven`, `g` is `isOdd`

Scope of variables

- Is this allowed?

```haskell
haskellIsAwesome = True

haskellIsAwesome = False -- changed my mind
```

- Answer: no, a variable can be defined once per scope; no mutation!

Local variables

- You can introduce a *new* (local) scope using a `let`-expression

```
sum 0 = 0
sum n = let n' = n - 1
        in n + sum n'   -- the scope of n'
                        -- is the term after in
```

- Syntactic sugar for nested `let`-expressions:

```
sum 0 = 0
sum n = let
          n'   = n - 1
          sum' = sum n'
        in n + sum'
```

Local variables

- If you need a variable whose scope is an equation, use the `where` clause instead:

```
cmpSquare x y  |  x > z   =  "bigger :)"
               |  x == z  =  "same :|"
               |  x < z   =  "smaller :("
  where z = y*y
```

Types

- What would *Elsa* say?

```
let FNORD = ONE ZERO
```

- **Answer**: Nothing. When evaluated, it will crunch to *something*, but it will be nonsensical.
  - λ-calculus is **untyped**.

Types

- What would *Python* say?

```
def fnord():
  return 0(1)
```

- **Answer**: Nothing. When evaluated will cause a run-time error.
  - Python is **dynamically typed**

Types

- What would *Java* say?

```
void fnord() {
  int zero;
  zero(1);
}
```

- **Answer**: Java compiler will reject this.
  - Java is **statically typed**.

Types

- In *Haskell* every expression either **has a type** or is **ill-typed** and rejected statically (at compile-time, before execution starts)
  - like in Java
  - unlike λ-calculus or Python

```
fnord = 1 0      -- rejected by GHC
```

Type Annotations

- You can annotate your bindings with their types using `::`, like so:

```haskell
-- | This is a Boolean:
haskellIsAwesome :: Bool
haskellIsAwesome = True

-- | This is a string
message :: String
message = if haskellIsAwesome
            then "I love CMPS 112"
            else "I'm dropping CMPS 112”
```

Type Annotations

```haskell
-- | This is a word-size integer
rating :: Int
rating = if haskellIsAwesome then 10 else 0

-- | This is an arbitrary precision integer
bigNumber :: Integer
bigNumber = factorial 100
```

- If you omit annotations, GHC will infer them for you
  - Inspect types in GHCi using `:t`
  - You should annotate all top-level bindings anyway! (Why?)

Function Types

- Functions have **arrow types**
  - `\x -> e` has type `A -> B`
  - If `e` has type `B`, assuming `x` has type `A`
- For example:

```
> :t (\x -> if x then 'a' else 'b')
(\x -> if x then 'a' else 'b') :: Bool -> Char
```

Function Types

- You should annotate your function bindings:

```
sum :: Int -> Int
sum 0 = 0
sum n = n + sum (n - 1)
```

- With multiple arguments:

```
pair :: String -> (String -> (Bool -> String))
pair x y b = if b then x else y
```

- Same as:

```
pair :: String -> String -> Bool -> String
pair x y b = if b then x else y
```

QUIZ: Type of Pair

With `pair :: String -> String -> Bool -> String`, what would GHCi say

```
>:t pair "apple" "orange"
```

A. Syntax error

B. The term is ill-typed

C. `String`

D. `Bool -> String`

E. `String -> String -> Bool -> String`

http://tiny.cc/cse116-tpair-ind

QUIZ: Type of Pair

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C. `String`

D. `Bool -> String`

E. `String -> String -> Bool -> String`

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Lists

- A list is
  - either an *empty list*

```
[] -- pronounced "nil"
```

  - or a *head element* attached to a *tail list*

```
x:xs -- pronounced "x cons xs"
```

Terminology: constructors and values

```
[]                -- A list with zero elements

1:[]              -- A list with one element: 1

(:) 1 []          -- Same thing: for any infix op,
                  -- (op) is a regular function!

1:(2:(3:(4:[])))  -- A list with four elements: 1, 2, 3, 4

1:2:3:4:[]        -- Same thing (: is right associative)

[1,2,3,4]         -- Same thing (syntactic sugar)
```

Lists

- `[]` and `(:)` are called the list **constructors**
- We've seen constructors before:
  - `True` and `False` are `Bool` constructors
  - `0`, `1`, `2` are… well, it's complicated, but you can think of them as `Int` constructors
  - these constructions didn't take any parameters, so we just called them *values*
- In general, a **value** is a constructor applied to *other values* (e.g., *list values* on previous slide)

Type of a list

- A list has type `[A]` if each one of its elements has type `A`
- Examples:

```
myList :: [Int]
myList = [1,2,3,4]

myList' :: [Char]                     -- or :: String
myList' = ['h', 'e', 'l', 'l', 'o'] -- or = "hello"

myList'' = [1, ‘h'] -- Type error: elements have
                    --  different types!

myList''' :: [t] -- Generic: works for any type t!
myList''' = []
```

Functions on lists: range

```
-- List of integers from n upto m
upto :: Int -> Int -> [Int]
upto n m  | n > m      = []
          | otherwise = n : (upto (n + 1) m)
```

- There is also syntactic sugar for this!

```
[1..7]     -- [1,2,3,4,5,6,7]
[1,3..7]   -- [1,3,5,7]
```

Functions on lists: length

```
-- Length of the list
length :: ???
length xs = ???
```

Pattern matching on lists

```
-- Length of the list
length :: [Int] -> Int
length []      =
length (x:xs)  =
```

Pattern matching on lists

```haskell
-- Length of the list
length :: [Int] -> Int
length []     = 0
length (_:xs) = 1 + length xs
```

- ~~A pattern is either a *variable* (incl. _) or a *value*~~
- A pattern is
  - either a *variable* (incl. _)
  - or a *constructor* applied to other *patterns*
- **Pattern matching** attempts to match *values* against *patterns* and, if desired, *bind* variables to successful matches.

QUIZ: Patterns

Which of the following is not a pattern? *

- A. (1 : xs)
- B. (_ : _ : _)
- C. [x]
- D. [1+2, x, y]
- E. all of the above

http://tiny.cc/cse116-pattern-ind

QUIZ: Patterns (wrong url)

Which of the following is not a pattern? *

- A. (1 : xs)
- B. (_ : _ : _)
- C. [x]
- D. [1+2, x, y]
- E. all of the above

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Some useful library functions

```haskell
-- | Is the list empty?
null :: [t] -> Bool

-- | Head of the list
head :: [t] -> t    -- careful: partial function!

-- | Tail of the list
tail :: [t] -> [t] -- careful: partial function!

-- | Length of the list
length :: [t] -> Int

-- | Append two lists
(++) :: [t] -> [t] -> [t]

-- | Are two lists equal?
(==) :: [t] -> [t] -> Bool
```

You can search for library functions (by type!) at <u>hoogle.haskell.org</u>

Pairs

```haskell
myPair :: (String, Int)  -- pair of String and Int
myPair = ("apple", 3)
```

- `(,)` is the pair constructor

```haskell
-- Field access using library functions:
fst myPair      -- "apple"

isEmpty p       =  (snd p) == 0

-- OR: Field access using pattern matc
isEmpty (x, y)  =  y == 0

isEmpty         = \(x, y) -> y == 0

isEmpty p       = let (x, y) = p in y
```

You can use pattern matching not only in equations, but also in λ-bindings and `let`-bindings!

Pattern matching with pairs

- Is this pattern matching correct? What does this function do?

```
f :: String -> [(String, Int)] -> Int
f _ []    = 0
f x ((k,v) : ps)
  | x == k    = v
  | otherwise = f x ps
```

Pattern matching with pairs

- Is this pattern matching correct? What does this function do?

```
f :: String -> [(String, Int)] -> Int
f _ []    = 0
f x ((k,v) : ps)
  | x == k     = v
  | otherwise = f x ps
```

- ***Answer***: a list of pairs represents key-value pairs in a dictionary; `f` performs lookup by key

Tuples

- Can we implement triples like in λ-calculus?
- Sure! But Haskell has native support for *n*-tuples:

```
myPair   :: (String, Int)
myPair   = ("apple", 3)

myTriple :: (Bool, Int, [Int])
myTriple = (True, 1, [1,2,3])

my4tuple :: (Float, Float, Float, Float)
my4tuple = (pi, sin pi, cos pi, sqrt 2)

...
-- And also:
myUnit   :: ()
myUnit   = ()
```

List comprehensions

- A convenient way to construct lists from other lists:

```
[toUpper c | c <- s]  -- Convert string s to upper case



[(i,j) | i <- [1..3],
         j <- [1..i] ] -- Multiple generators

[(i,j) | i <- [0..5],
         j <- [0..5],
         i + j == 5] -- Guards
```

Quicksort in Haskell

```
sort []     = []
sort (x:xs) = sort ls ++ [x] ++ sort rs
  where
    ls      = [ l | l <- xs, l <  x ]
    rs      = [ r | r <- xs, x >  r ]
```

What is Haskell?

- A **typed**, **lazy**, **purely functional** programming language

Haskell is statically typed

- Every expression either has a type, or is *ill-typed* and rejected at compile time
- **Why is this good?**
  - catches errors early
  - types are contracts (you don't have to handle ill-typed inputs!)
  - enables compiler optimizations

Haskell is purely functional

- **Functional** = functions are *first-class values*
- **Pure** = a program is an expression that evaluates to a value
  - No side effects! unlike in Python, Java, etc:

```
public int f(int x) {
  calls++;                         // side effect!
  System.out.println("calling f"); // side effect!
  launchMissile();                 // side effect!
  return x * 2;
}
```

  - in Haskell, a function of type `Int -> Int` computes a *single integer output* from a *single integer input* and does **nothing else**

Haskell is purely functional

- **Referential transparency:** The same expression always evaluates to the same value
  - More precisely: In a scope where `x1, ..., xn` are defined, all occurrences of `e` with `FV(e) = {x1, ..., xn}` have the same value
- **Why is this good?**
  - easier to reason about (remember `x++` vs `++x` in C?)
  - enables compiler optimizations
  - especially great for parallelization (`e1 + e2`: we can always compute `e1` and `e2` in parallel!)

Haskell is lazy

- An expression is evaluated only when its result is needed
- **Example:** `take 2 [1 .. (factorial 100)]`

```
        take 2 (    upto 1 (factorial 100))
=>      take 2 (    upto 1 933262154439...)
=>      take 2 (1:(upto 2 933262154439...)) -- def upto
=> 1:   (take 1 (    upto 2 933262154439...)) -- def take 3
=> 1:   (take 1 (2:(upto 3 933262154439...)) -- def upto
=> 1:2:(take 0 (    upto 3 933262154439...)) -- def take 3
=> 1:2:[]                                     -- def take 1
```

–

Haskell is lazy

- **Why is this good?**
  - Can implement cool stuff like infinite lists: `[1..]`

```
-- first n pairs of co-primes:
take n [(i,j) | i <- [1..],
                j <- [1..i],
                gcd i j == 1]
```

  - encourages simple, general solutions
  - but has its problems too :(

That’s all folks!