

# Lecture 05.29.26. FRIDAY

## - Announcements

- Posted the slides link on Ed Discussion
- I will post my notes later today
- Next week? Post a poll of what you want for last lectures
- Assignment due Saturday

## - Agenda

- Finish up unification
- Type inference algorithm.

## \* UNIFICATION

- Type variables,  $a, b, c$
- Type substitution  
 $[a/Int, b/c \rightarrow c]$  substitution
- Applying a substitution  
 $[a/Int, b/c \rightarrow c]$   
 $a \rightarrow a \quad Int \rightarrow Int$
- If there are no type variables in the type that the substitution is being applied to, then it stays the same.

$$a \rightarrow d \quad Int \rightarrow d$$

## \* UNIFIERS

$$\left. \begin{array}{l} a \rightarrow Int \\ Bool \rightarrow b \end{array} \right\} [a/Bool, b/Int]$$

$$Bool \rightarrow Int$$

? Question?

~~$[Bool/a, Int/b]$~~  ?

- This is not a substitution
- A substitution only maps type variables ( $a, b, c, \dots$  etc.) to types
- $Bool + Int$  are not type variables so they can't go on LHS
- Caveats
  - If the types I have are the same to begin with, then an empty substitution will unify them.  $a \rightarrow b, a \rightarrow b \quad []$
  - $a, b$  (type variables) and  $a \rightarrow b$  different then  $[a/b,] + [b/a]$  are unifiers.
  - If I have type variable + another type.  $a \rightarrow Int \rightarrow Bool$  then  $[a/Int \rightarrow Bool]$

? Question?

- Suppose I have a  
 $\text{Int} \rightarrow a$

Can I unify these? Why or why not?

- Yes?

- No?

$[a / \text{Int} \rightarrow a]$  ? No

- If we apply this  $a$  to

$\text{Int} \rightarrow a$

- If we apply this to  $\text{Int} \rightarrow a$

$\text{Int} \rightarrow \text{Int} \rightarrow a$

\* You can't unify a variable with any type that contains free occurrences of that variable.

- "occurs check"

- Question

can we use  $\alpha$ -step for this

Add my slides.

- Examples

- |    |   |   |   |
|----|---|---|---|
| 1  | - $a + \text{Int}$                                      | ✓ | $[a / \text{Int}]$                        |
| 2  | - $a \rightarrow a + \text{Int} \rightarrow \text{Int}$ | ✓ | $[a / \text{Int}]$                        |
| 3  | - $a + \text{Int} \rightarrow \text{Int}$               | ✓ | $[a / \text{Int} \rightarrow \text{Int}]$ |
| 4  | - $a + b \rightarrow c$                                 | ✓ | $[a / b \rightarrow c]$                   |
| 5  | - $a \rightarrow \text{Int} + \text{Int} \rightarrow b$ | ✓ | $[a / \text{Int}, b / \text{Int}]$        |
| 6  | - $\text{Int} + \text{Int}$                             | ✓ | $[\ ]$                                    |
| 7  | - $a + a$   | ✓ | $[\ ]$                                    |
| 8  | - $a + b$   | ✓ | $[a / b]$                                 |
| 9  | - $\text{Int} + \text{Int} \rightarrow \text{Int}$      | ✗ |   |
| 10 | - $\text{Int} + a \rightarrow a$                        | ✗ |   |