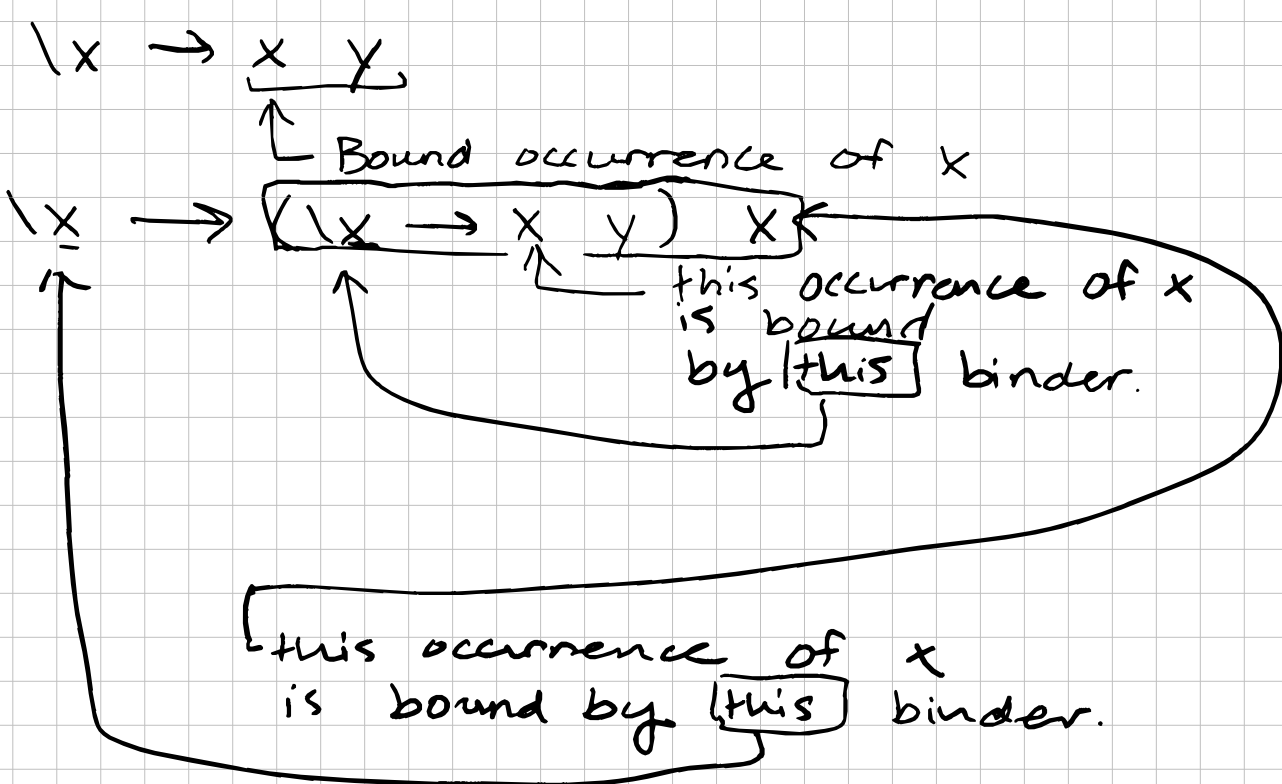
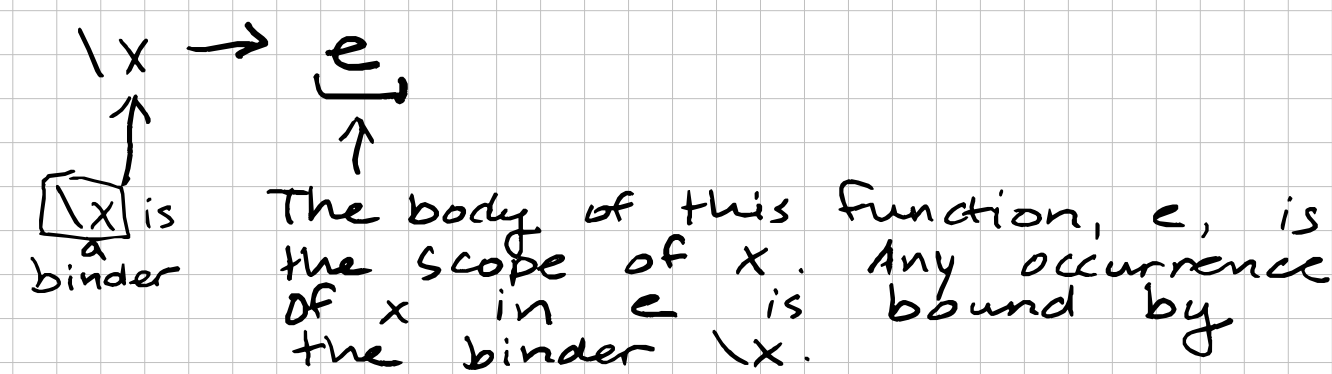


CSE 114A Lecture 4

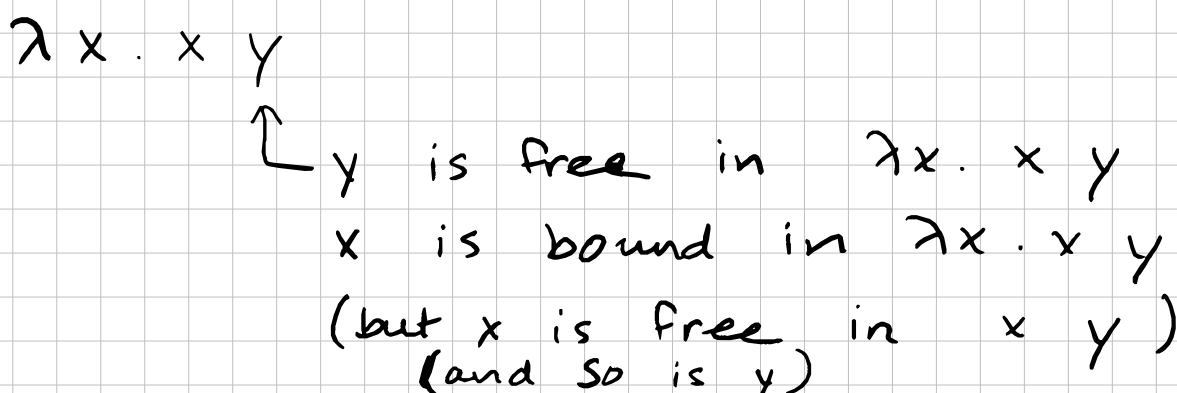
- ✓ - announcement
- wrap-up of lambda calc!
- ✓ - Scope of a variable; renaming
- ✓ - Normal form; nontermination (omega)
- pairs (structs, tuples, ...)
- Why is ADD defined how it is?
- Recursion!
- zooming out: the big picture. Why discuss λ -calc?

Scope and renaming.

Scope of a variable: the part of a program where the variable is visible.



An occurrence of a variable is free if it's not bound by an enclosing lambda abstraction.



Renaming $e ::= x / \lambda x. e / e_1, e_2$

first let's define what the free variables of an arbitrary λ -calc expression are

- $FV(x) = \{x\}$
- $FV(\lambda x. e) = FV(e) - \{x\}$ (set difference)
- $FV(e_1, e_2) = FV(e_1) \cup FV(e_2)$

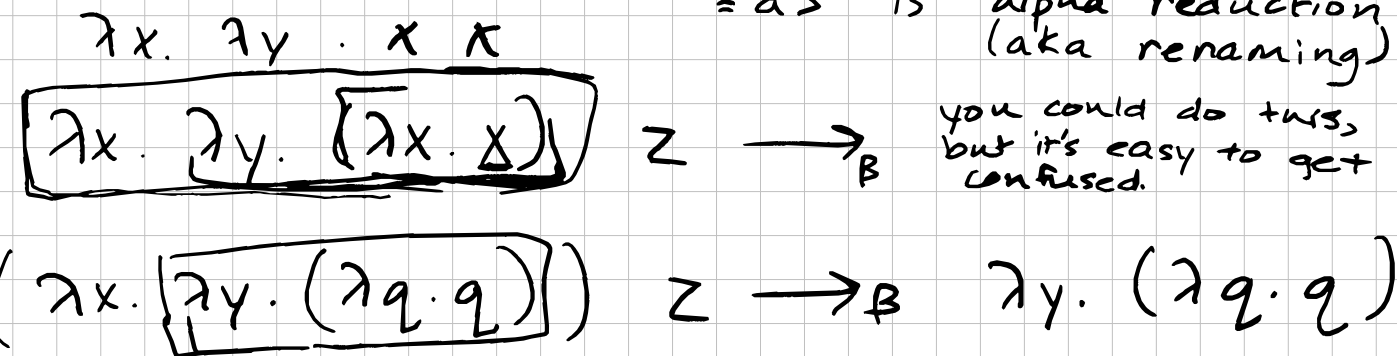
This gives us a policy for how to do renaming!

$$\lambda x. e \xrightarrow{\alpha} \lambda y. e[x := y]$$

where y is not in $FV(e)$

$$\lambda x. x = \lambda y. y = \lambda z. z, \text{ for example.}$$

In Elsa, much like $=b>$ is beta reduction, (aka substitution), $=a>$ is alpha reduction (aka renaming).



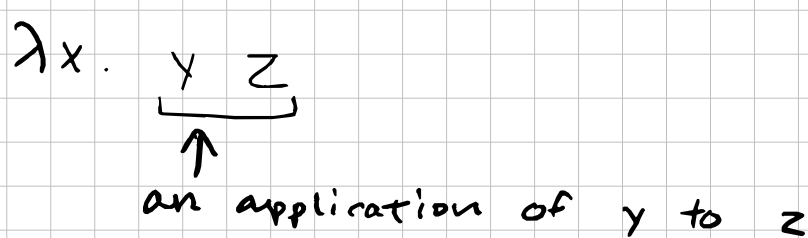
Normal form

a "reducible expression" (or redex for short): is an expression of the form $(\lambda x. e_1) e_2$ (in other words, the left side of the substitution rule!)

If you have any redexes anywhere in a λ -calc expression, it's not in normal form.

If you have no redexes, it's in normal form.

An expression in normal form is a value (and can't be further reduced).



$$(\lambda y z \rightarrow (\lambda x \rightarrow y z)) \text{ bulbasaur mew}$$

$=b>$

$$(\lambda z \rightarrow (\lambda x \rightarrow \text{bulbasaur } z)) \text{ mew}$$

$=b>$

$$\lambda x \rightarrow \text{bulbasaur mew}$$

This might eventually be a redex, but right now it can't be reduced.

Do all λ -calc expressions have a normal form?
(Do they all evaluate to a value?)

No!

$(\lambda x. \boxed{x x}) (\lambda x. x x)$

x applied
to x

The omega
combinator.

\rightarrow_B

$(\lambda x. x x) (\lambda x. x x)$ oh no!

We'll never get to a value.