

# CSE114A, Fall 2023: Midterm Exam

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This exam has 5 questions and 95 total points.

## Instructions

- Please write directly on the exam.
- For multiple choice questions, **fill in the letter completely**, e.g. from (a) to ●
- For short response questions, try to keep your answer within the outlined box.
- **You have 65 minutes to complete this exam.** You may leave when you are finished.
- This exam is **closed book**. You may use one double-sided page of notes, but no other materials.
- Avoid seeing anyone else’s work or allowing yours to be seen.
- Please, no talking. No notes, books, laptops, phones, or other electronic devices. Do not communicate with anyone but an exam proctor.
- To ensure fairness (and the appearance thereof), **proctors will not answer questions about the content of the exam**. If you are unsure of how to interpret a problem description, state your interpretation clearly and concisely. *Reasonable interpretations* will be taken into account by graders.

Good luck!

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## Part 1: Lambda calculus

### Question 1 (15 points)

Consider the following lambda calculus expression, which we will name `EXPR1`:

```
(\x y -> ITE b (ADD x y)
  ((\z -> ITE (ISZ z) x (ADD x z)) FALSE)) FOUR ONE
```

1.1 (5 points) What are the free variables of `EXPR1`?

- (a) `x`
- (b) `x, y`
- (c) `b`
- (d) Choices (b) and (c)
- (e) None of the above

1.2 (5 points) After a *single*  $\beta$ -reduction step on `EXPR1`, what would the resulting expression be?

- (a) 

```
(\y -> ITE b (ADD FOUR y)
  ((\z -> ITE (ISZ z) FOUR (ADD FOUR z)) FALSE)) ONE
```
- (b) 

```
(\y -> ITE b (ADD FOUR y)
  (\z -> ITE (ISZ z) FOUR (ADD FOUR z))) ONE
```
- (c) 

```
(\x y -> ITE b (ADD x y)
  (ITE (ISZ FALSE) x (ADD x FALSE))) FOUR ONE
```
- (d) Choices (a) and (b)
- (e) Choices (a) and (c)

1.3 (5 points) What is the *normal form* of `EXPR1`?

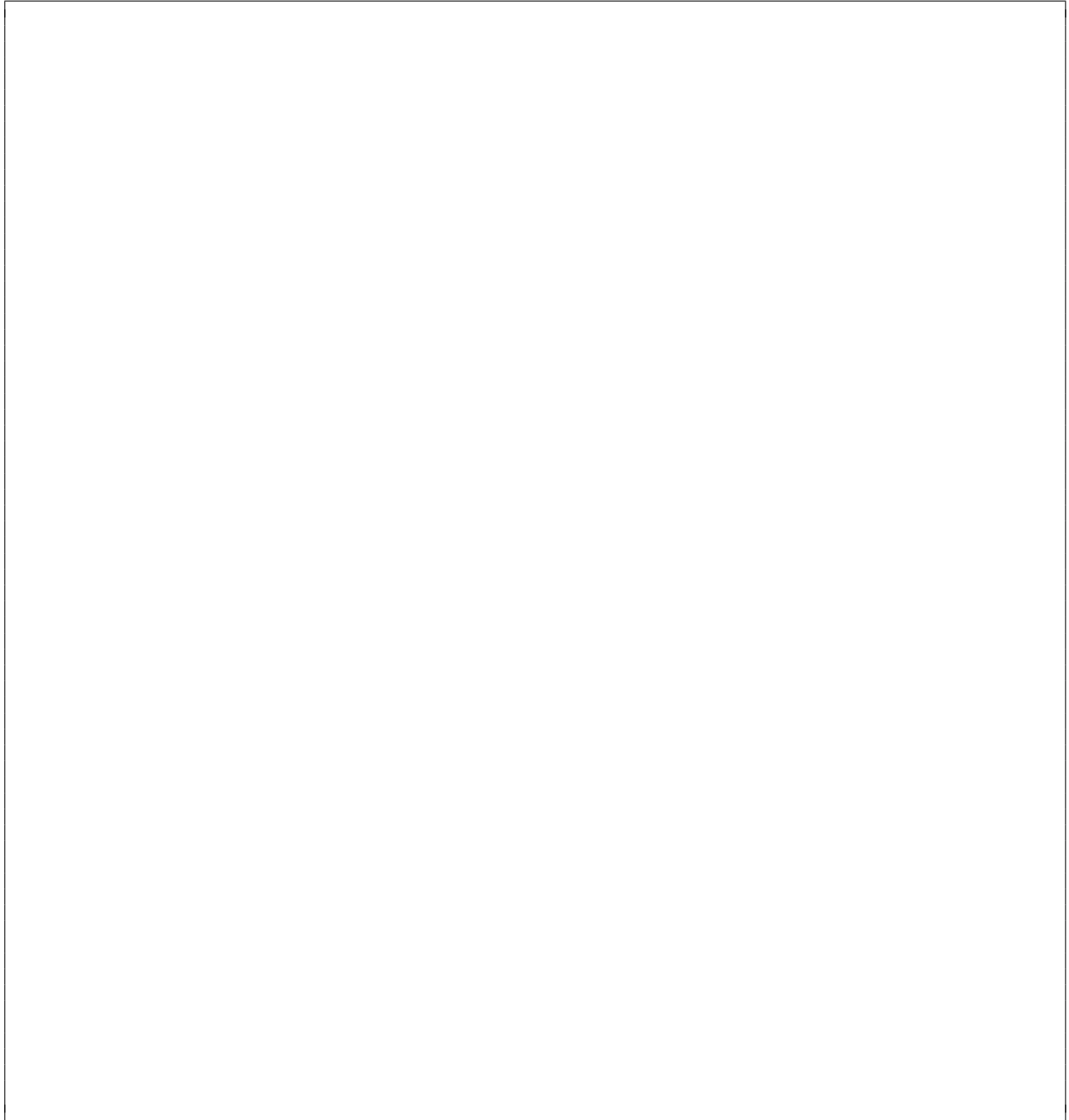
- (a) `FOUR`
- (b) `FIVE`
- (c) `b FIVE FOUR`
- (d) 

```
(\z -> ITE (ISZ z) FOUR (ADD FOUR z))
```
- (e) None of the above

Question 2 (15 points)

Reduce the following expression to normal form step-by-step using =a>, =b>, and =d>.

`(\x y -> ITE (ISZ y) x (ADD x y)) FIVE ZERO`



Question 3 (15 points)

Consider the following Haskell function:

```
listFun xs = helper ([], []) xs
  where
    helper (arr1, arr2) [] = (arr1, arr2)
    helper (arr1, arr2) (x:xs)
      | x `mod` 2 == 0 = helper (arr1 ++ [x], arr2) xs
      | otherwise = helper (arr1, arr2 ++ [x]) xs
```

3.1 (5 points) What is the type of listFun?

- (a) Int -> [Int]
- (b) [Int] -> [Int]
- (c) Int -> (Int, Int)
- (d) [Int] -> ([Int], [Int])
- (e) None of the above

3.2 (5 points) What does listFun [1, 4, 9, 12, 21, 33, 4, 52] evaluate to?

- (a) [1, 4, 4, 9, 12, 21, 33, 52]
- (b) 149122133452
- (c) ([52, 4, 12, 4], [33, 21, 9, 1])
- (d) ([4, 12, 4, 52], [1, 9, 21, 33])
- (e) None of the above

3.3 (5 points) Is listFun tail recursive?

- (a) No, but a tail recursive implementation **is possible**.
- (b) No, but a tail recursive implementation **is not possible**.
- (c) Yes

Question 4 (28 points)

Consider the binary search tree below where each internal node has a value  $i$  and two child subtrees. The left child contains all nodes with values less than or equal to  $i$ , and the right child contains all nodes with values strictly greater than  $i$ .

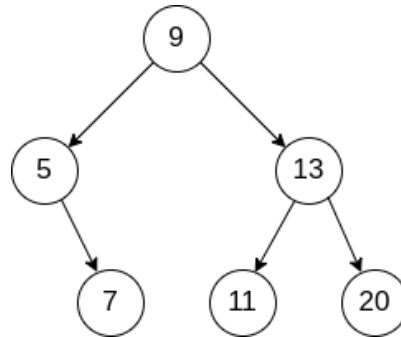


Figure 1: A binary search tree

Given the following ADT that is used to represent the tree above, answer the questions below. **You may not use library functions except for (+) and (++)**.

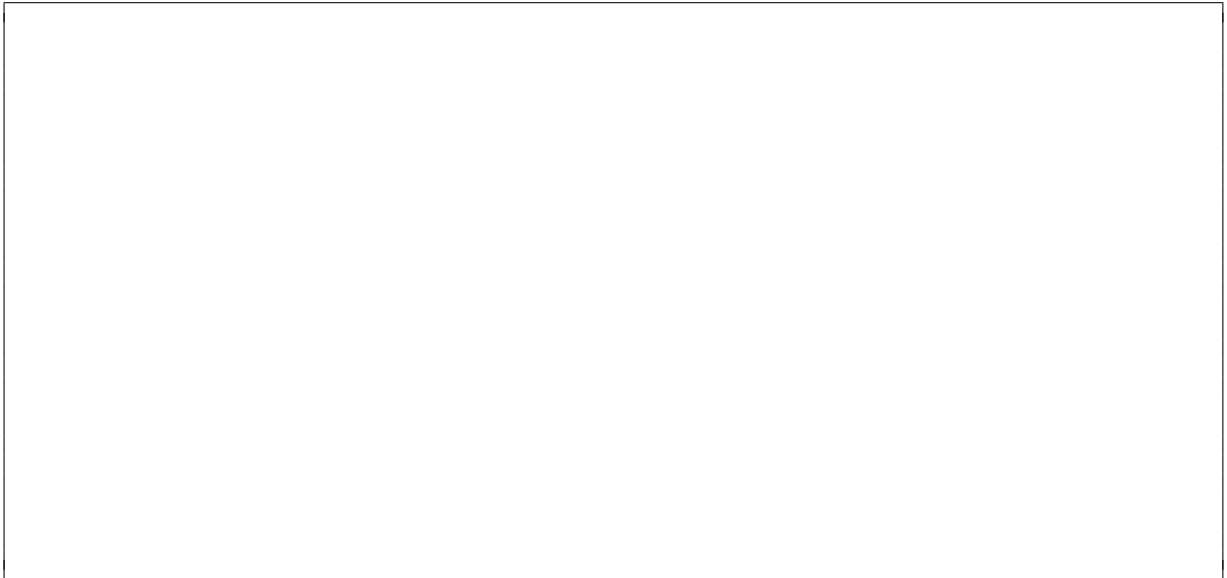
```
data Tree = Leaf | Node Int Tree Tree
```

4.1 (7 points) Define a function `sumTree`, which returns the sum of all of the values in the nodes within a tree *without using tail recursion*. For example, if `t` is the tree in the figure above, then `sumTree t` evaluates to 65.

```
sumTree :: Tree -> Int
```

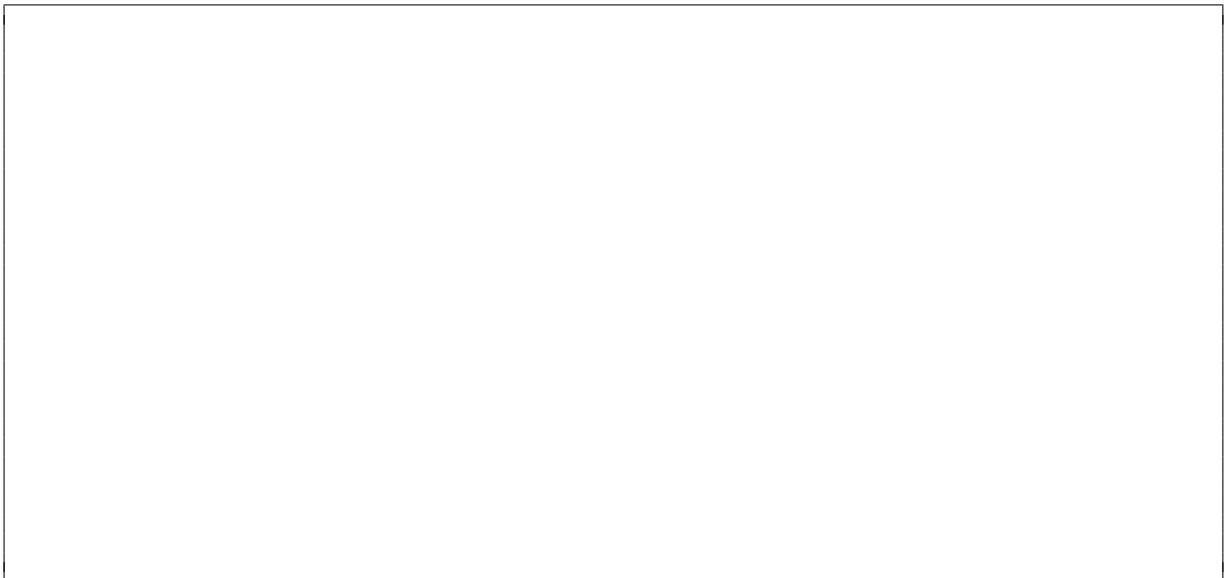
- 4.2 (7 points) Define a function `sumTreeTR`, which returns the sum of all the values in the nodes within a tree *using tail recursion*. For example, if `t` is the tree in the figure above, then `sumTree t` evaluates to 65.

```
sumTreeTR :: Tree -> Int
```



- 4.3 (7 points) Implement the function `inOrder`, which returns the list of nodes from smallest to largest. For example, if `t` is the tree in the figure above, then `inOrder t` evaluates to `[5, 7, 9, 11, 13, 20]`. (It may be head or tail recursive.)

```
inOrder :: Tree -> [Int]
```



4.4 (7 points) Now use `inOrder` to implement `sumTree` and `sumTreeTR` using `foldr` and `foldl`. Your implementation should use the correct fold for head or tail recursion. The implementation of `inOrder` above does not matter, even if incomplete.





Question 5 (22 points)

Haskell has some functions that let you convert Char values to Int values and vice-versa. Function `ord :: Char -> Int` converts a Char to an Int corresponding to its position in the alphabet (starting at 0), so (`ord 'a'`) is 0 and (`ord 'z'`) is 25. Function `chr :: Int -> Char` converts a Int to the Char corresponding to the letter at that position (mod 26) in the alphabet. So (`chr 0`) is 'a', (`chr 25`) is 'z', and (`chr 26`) is 'a'.

Consider the specifications for the higher-order functions `enc` and `dec`.

```
-- 'enc f txt' applies f to each character of 'txt' to encode a string.
enc :: (Char -> Char) -> String -> String
-- 'dec g txt' applies g to each character of 'txt' to decode 'txt'.
dec :: (Char -> Char) -> String -> String
-- REQUIRED: if '(g (f c)) = c' then 'dec g (enc f str) = str'
```

5.1 (5 points) Given the above specifications, what does the following Haskell expression evaluate to?

```
let k c = chr ((ord c) + 13) in
  enc k "abc"
```

- Ⓐ "xyz"
- Ⓑ "nmo"
- Ⓒ "abc"
- Ⓓ Type Error
- Ⓔ None of the above

5.2 (5 points) Given the above specifications, what does the following Haskell expression evaluate to?

```
let k c = chr ((ord c) + 2) in
  dec k "rpgai"
```

- Ⓐ "trick"
- Ⓑ "treat"
- Ⓒ "candy"
- Ⓓ Type Error
- Ⓔ None of the above

5.3 (5 points) Given the above specifications, what does the following Haskell expression evaluate to?

```
let k n c = chr ((ord c) + n)
    k' n c = chr ((ord c) - n)
in
  dec (k' 5) (enc (k 5) "foo")
```

- Ⓐ "ktt"
- Ⓑ "ajj"
- Ⓒ "oof"
- Ⓓ "foo"
- Ⓔ Type Error

5.4 (7 points) Give an implementation for `enc` and `dec`. You may use any library functions you wish.

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# 1 Lambda calculus cheat sheet

-- Booleans -----

```
let TRUE = \x y -> x
let FALSE = \x y -> y
let ITE = \b x y -> b x y
let NOT = \b x y -> b y x
let AND = \b1 b2 -> ITE b1 b2 FALSE
let OR = \b1 b2 -> ITE b1 TRUE b2
```

-- Numbers -----

```
let ZERO = \f x-> x
let ONE = \f x -> f x
let TWO = \f x -> f (f x)
let THREE = \f x -> f (f (f x))
let FOUR = \f x -> f (f (f (f x)))
let FIVE = \f x -> f (f (f (f (f x))))
```

-- Pairs -----

```
let PAIR = \x y b -> b x y
let FST = \p -> p TRUE
let SND = \p -> p FALSE
```

-- Arithmetic -----

```
let INC = \n f x -> f (n f x)
let ADD = \n m -> n INC m
let MUL = \n m -> n (ADD m) ZERO
let ISZ = \n -> n (\z -> FALSE) TRUE
let DECR = \n -> -- decrement n by one --
let EQL = \a b -> -- return TRUE if a == b, otherwise FALSE --
```

-- Recursion -----

```
let FIX = \stp -> (\x -> stp (x x)) (\x -> stp (x x))
```

## 2 Haskell cheat sheet

```
data Maybe a = Nothing | Just a
```

```
foldr :: (a -> b -> b) -> b -> [a] -> b
```

```
foldr f b [] = b
```

```
foldr f b (x:xs) = f x (foldr f b xs)
```

```
foldl :: (b -> a -> b) -> b -> [a] -> b
```

```
foldl f b xs = helper b xs
```

```
  where
```

```
    helper acc [] = acc
```

```
    helper acc (x:xs) = helper (f acc x) xs
```

```
filter :: (a -> Bool) -> [a] -> [a]
```

```
filter p [] = []
```

```
filter p (x:xs)
```

```
  | p x = x : filter p xs
```

```
  | otherwise = filter p xs
```

```
map :: (a -> b) -> [a] -> [b]
```

```
map _ [] = []
```

```
map f (x:xs) = f x : map f xs
```

```
flip :: (a -> b -> c) -> b -> a -> c
```

```
flip f x y = f y x
```

```
(.) :: (b -> c) -> (a -> b) -> a -> c
```

```
(.) f g x = f (g x)
```

```
(++) :: [a] -> [a] -> [a]
```

```
(++) [] ys = ys
```

```
(++) (x:xs) ys = x : xs ++ ys
```

```
-- returns the elements of a list in reverse order.
```

```
reverse :: [a] -> [a]
```

```
-- Extract the first element of a list, which must be non-empty.
```

```
head :: [a] -> a
```

```
-- Extract the elements after the head of a list, which must be non-empty.
```

```
tail :: [a] -> [a]
```

```
-- Extract the first n elements of a list.
```

```
take :: Int -> [a] -> [a]
```

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