CSE114A, Fall 2022: Midterm Exam

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Student name: _____

CruzID (the part before the "@" in your UCSC email address): _____

This exam has 6 questions and 123 total points.

Instructions

- Please write directly on the exam.
- For multiple choice questions, **fill in the letter completely**, e.g. from (a) to ●
- **You have 95 minutes to complete this exam.** You may leave when you are finished.
- This exam is **closed book**. You may use one double-sided page of notes, but no other materials.
- Avoid seeing anyone else's work or allowing yours to be seen.
- Please, no talking. No notes, books, laptops, phones, or other electronic devices. Do not communicate with anyone but an exam proctor.
- To ensure fairness (and the appearance thereof), **proctors will not answer questions about the content of the exam**. If you are unsure of how to interpret a problem description, state your interpretation clearly and concisely. *Reasonable interpretations* will be taken into account by graders.

Good luck!

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Part 1: Lambda calculus

1. Consider the following lambda calculus expression, which we will name `EXPR1`:

```
(\b -> ITE b trick treat) (OR (\x y -> x) (\x y -> y)) candy prank
```

   a. (5 points) What are the free variables of `EXPR1`?
      (a) `b, x, y`
      (b) `candy, prank`
      (c) `trick, treat`
      (d) Choices (b) and (c)
      (e) Choices (a) and (b)

   b. (5 points) After a *single* β-reduction step on `EXPR1`, what would the resulting expression be?
      (a) `(\b -> ITE b trick treat) (OR candy (\x y -> y)) prank`
      (b) `(ITE (OR (\x y -> x) (\x y -> y)) trick treat) candy prank`
      (c) `(\b -> ITE b trick treat) (\x y -> x) candy prank`
      (d) Choices (b) or (c)
      (e) None of the above

   c. (5 points) What is the *normal form* of `EXPR1`?
      (a) `trick`
      (b) `treat`
      (c) `treat candy`
      (d) `trick candy prank`
      (e) None of the above

2. Consider the following lambda calculus expression, which we will name `EXPR2`:

```
let F = (\rec l -> rec (PAIR (PLUS (FST l) (FST (SND l))) l))
(FIX F) (PAIR ONE (PAIR ZERO FALSE))
```

   a. (5 points) What does `EXPR2` evaluate to?
      (a) `(PAIR TWO (PAIR ONE (PAIR ONE (PAIR ZERO FALSE))))`
      (b) `F (FIX F) (PAIR ONE (PAIR ZERO FALSE))`
      (c) `STEP (FIX STEP)`
      (d) `EXPR2` does not have a normal form.
      (e) `EXPR2` does not type check.

   b. (5 points) Which of the following reductions of `EXPR2` is valid? (Recall `=*>` denotes any sequence of `=a>`, `=b>`, or `=d>` reductions.)
      (a)
```
=*> F (F (F (FIX F)))
      (PAIR TWO (PAIR ONE (PAIR ONE (PAIR ZERO FALSE))))
```
      (b)
```
=*> (FIX F)
      (PAIR TWO (PAIR ONE (PAIR ONE (PAIR ZERO FALSE))))
```
      (c)
```
=*> F (FIX F)
      (PAIR FIVE (PAIR THREE
       (PAIR TWO (PAIR ONE (PAIR ONE (PAIR ZERO FALSE))))))
```
      (d) All of the above.
      (e) None of the above.

Part 2: Haskell

3. (8 points) What does the following Haskell expression evaluate to?

```
let
    f x = x + 1
    g = \f x -> f (f x)
    h f = \y -> y (y f)
    i k = k
    k = \a b -> a i b
in
  k g (flip h) f 0
```

   (a) 0

   (b) 2

   (c) 4

   (d) Infinite loop

   (e) Type Error

4. Consider the following Haskell function:

```
mkpal s = s ++ (helper s)
  where
    helper []     = ""
    helper (c:cs) =  (helper cs) ++ [c]
```

   a. (5 points) What is the type of `mkpal`?

      (a) `String -> [String]`

      (b) `[String] -> [String]`

      (c) `String -> String`

      (d) `[String] -> [Char]`

      (e) None of the above

   b. (5 points) What does `mkpal "foobarbaz"` evaluate to?

      (a) `"foobarbaz"`

      (b) `"zabraboof"`

      (c) `"foobarbazbazbarfoo"`

      (d) `"foobarbazzabraboof"`

      (e) None of the above

   c. (5 points) Is `mkpal` tail recursive?

      (a) Yes

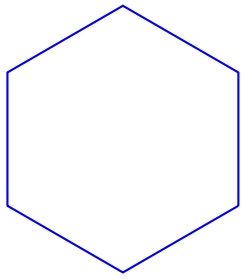
      (b) No, but a tail recursive implementation **is not possible**.

      (c) No, but a tail recursive implementation **is possible**.

5. *Turtle graphics*[1] are an approach to drawing figures by directing a cursor (or "turtle") around a canvas using simple set of commands. For example, the program below would draw a (regular) hexagon with sides of length 50 and internal angles of 120 degrees since turning the turtle left 60 degrees creates an angle of $180 - 60 = 120$.

```
repeat 6 {
  forward 50
  rotate_left 60
}
```

(a) Turtle program



(b) Program output

For the next questions, consider the following definitions that describe a simple language for turtle graphics.

```
data TurtleCmd = Home                -- return to starting position and orientation
         | Forward Int               -- move turtle forward by Int pixels
         | Back Int                  -- move turtle forward by Int pixels
         | RotateL Int               -- rotate left by Int degrees
         | RotateR Int               -- rotate right by Int degrees
         | Repeat Int [TurtleCmd]    -- repeat a list of commands Int times
type Plan = [TurtleCmd]
```

a. (5 points) In the box below, write a `Plan` corresponding to the hexagon-drawing program above.

[1] Turtle graphics were a central part of the *Logo* programming language. Look it up!

b. (20 points) Now help complete an interpreter for a `Plan` program. For each basic TurtleCmd there is a `do` function that applies the command to a `Canvas` value representing the state of the drawing and turtle. You don't have to worry about the definition of `Canvas`, just that you must provide the current canvas to each `do` function. You may use the second box for any helper functions.

Hint: Note that there is (intentionally) no `do` function for `Repeat`: you must write Haskell code to evaluate `Repeat` commands. You do not need the definition of `Canvas` to accomplish this.

```
doHome     :: Canvas -> Canvas
doForward  :: Canvas -> Int -> Canvas
doBack     :: Canvas -> Int -> Canvas
doRotateL  :: Canvas -> Int -> Canvas
doRotateR  :: Canvas -> Int -> Canvas

eval :: Canvas -> Plan -> Canvas
eval c [] =                          -- fill in the base case here

eval c (cmd:rest) =
  let c' = case cmd of -- fill in the body of the case function below
```

```
  in
    eval c' rest   -- use the below box for helper functions or where clauses
```

6. (25 points) Consider the following higher-order function `gen`:

```
gen :: (b -> Maybe (a, b)) -> b -> [a]
gen f b0 =
  let go b = case f b of
               Just (a, new_b) -> a : go new_b
               Nothing         -> []
  in go b0
```

a. (5 points) Choose the best answer:
   (a) `gen` always terminates (on any input)
   (b) `gen` never terminates (on any input)
   (c) `gen` may or may not terminate
   (d) `gen` terminates if and only if `f b` terminates
   (e) None of the above

b. (5 points) What does
   `gen (\b if b >= 5 then Nothing else Just (b,b+1)) 10` evaluate to?
   (a) [10,9,8,7,6]
   (b) [6,7,8,9,10]
   (c) []
   (d) Does not type check
   (e) Does not terminate

c. (5 points) What does
   `gen (\b if b >= 10 then Nothing else Just (b,b+1)) 5` evaluate to?
   (a) [9,8,7,6,5]
   (b) [5,6,7,8,9]
   (c) []
   (d) Does not type check
   (e) Does not terminate

d. (5 points) What does
   `take 5 (gen (\b if b < 5 then Nothing else Just (b,b+1)) 10)` evaluate to?
   (a) [5,6,7,8,9]
   (b) [10,11,12,13,14]
   (c) []
   (d) Does not type check
   (e) Does not terminate

e. (5 points) What does

```
helper []     = Nothing
helper (x:xs) = Just (x:xs)
gen helper (foldr (:) [] [6,7,8,9,10])
```

   evaluate to?
   (a) [10,9,8,7,6]
   (b) [6,7,8,9,10]
   (c) []
   (d) Does not type check
   (e) Does not terminate

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1 Lambda calculus cheat sheet

```
-- Booleans --------------------------------
let TRUE =\x y -> x
let FALSE = \x y -> y
let ITE = \b x y -> b x y
let NOT = \b x y -> b y x
let AND = \b1 b2 -> ITE b1 b2 FALSE
let OR = \b1 b2 -> ITE b1 TRUE b2

-- Numbers --------------------------------
let ZERO = \f x-> x
let ONE = \f x -> f x
let TWO = \f x -> f (f x)
let THREE = \f x -> f (f (f x))
let FOUR = \f x -> f (f (f (f x))
let FIVE = \f x -> f (f (f (f (f x))

-- Pairs ----------------------------------
let PAIR = \x y b -> b x y
let FST = \p -> p TRUE
let SND = \p -> p FALSE

-- Arithmetic -----------------------------
let INC   = \n f x -> f (n f x)
let ADD   = \n m -> n INC m
let MUL   = \n m -> n (ADD m) ZERO
let ISZ   = \n ->   -- return TRUE if n == 0 --
let DECR  = \n ->   -- decrement n by one --
let EQL   = \a b -> -- return TRUE if a == b, otherwise FALSE --

-- Recursion ------------------------------
let FIX = \stp -> (\x -> stp (x x)) (\x -> stp (x x))
```

2 Haskell cheat sheet

```haskell
data  Maybe a  =  Nothing | Just a

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f b []     = b
foldr f b (x:xs) = f x (foldr f b xs)

foldl :: (b -> a -> b) -> b -> [a] -> b
foldl f b xs          = helper b xs
  where
    helper acc []     = acc
    helper acc (x:xs) = helper (f acc x) xs

filter :: (a -> Bool) -> [a] -> [a]
filter p []    = []
filter p (x:xs)
  | p x         = x : filter p xs
  | otherwise      = filter p xs

map :: (a -> b) -> [a] -> [b]
map _ []     = []
map f (x:xs) = f x : map f xs

flip :: (a -> b -> c) -> b -> a -> c
flip f x y = f y x

(.) :: (b -> c) -> (a -> b) -> a -> c
(.) f g x = f (g x)

(++) :: [a] -> [a] -> [a]
(++) []     ys = ys
(++) (x:xs) ys = x : xs ++ ys

-- returns the elements of a list in reverse order.
reverse :: [a] -> [a]

-- Extract the first element of a list, which must be non-empty.
head :: [a] -> a

-- Extract the elements after the head of a list, which must be non-empty.
tail :: [a] -> [a]

-- Extract the first n elements of a list.
take :: Int -> [a] -> [a]
```

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